Using the Ensemble Transform Kalman Filter to estimate uncertainty in Full Waveform Inversion

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Goal of seismic tomography: find physical parameters of the subsurface from seismic wavefield data.

The recorded data are directly linked to the subsurface physical properties.

Comparison of observed wavefield with synthetic wavefield allows formulating an inverse problem to find the model that gives the best data-fit.

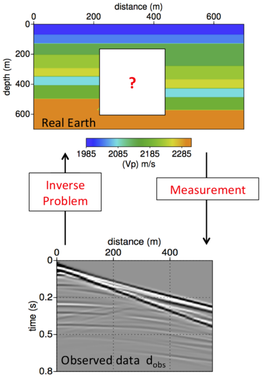
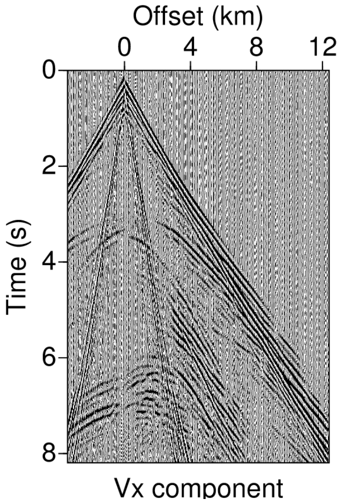


Illustration of the tomographic problem.

In Full Waveform Inversion (FWI), we try to match the entire recorded wavefield (d_{obs}) at receiver locations with the synthetic waveform data computed in a starting model (d_{cal}).

FWI allows to obtain higher resolution than "classical" tomography techniques relying only on travel time.

The FWI inverse problem is more difficult (more non-linear) as it attempts to fit an entire pressure recordings.



Exemple of recorded wavefield data.

Outline

Characteristics of FWI

From FWI to ETKF-FWI

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Conclusions

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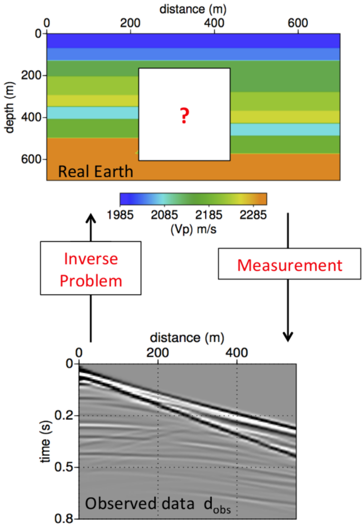
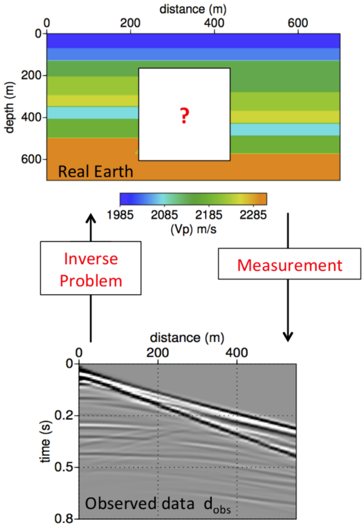
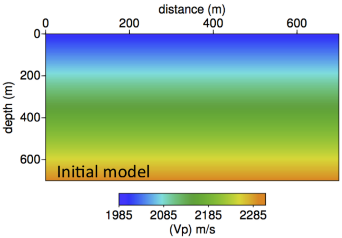
Courtesy of Isabella Masoni



Full Waveform Inversion Workflow

Courtesy of Isabella Masoni





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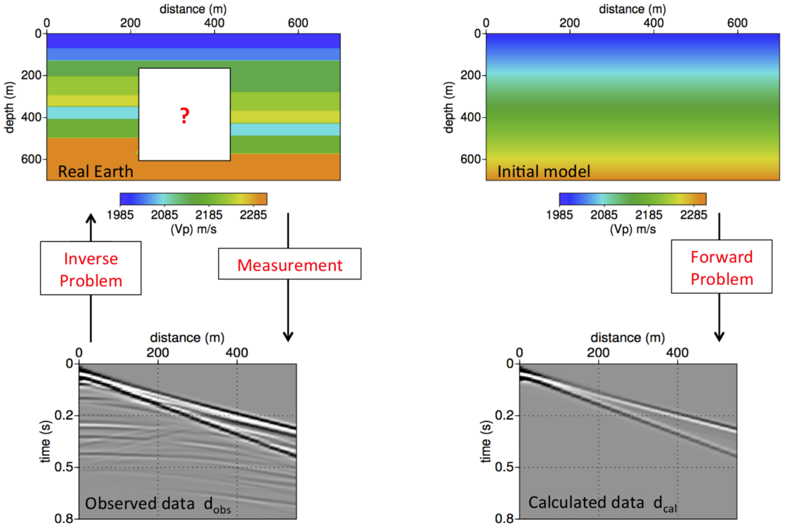
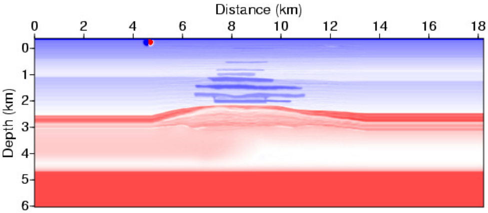


As we try to fit the recorded wavefield with synthetics, we need an appropriate forward modeling engine to reproduce accurately wave propagation physics.



The misfit of the FWI problem is : $\min_m \frac{1}{2}\|d_{cal}(m) - d_{obs}\|^2$

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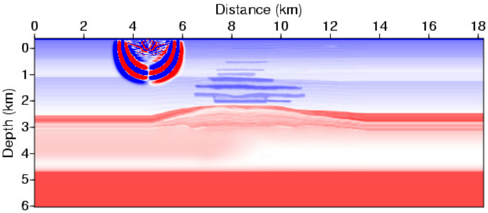

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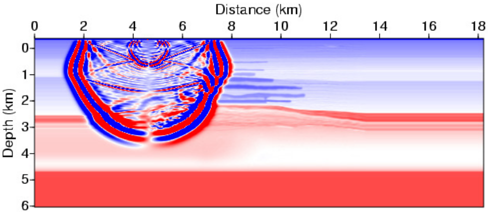

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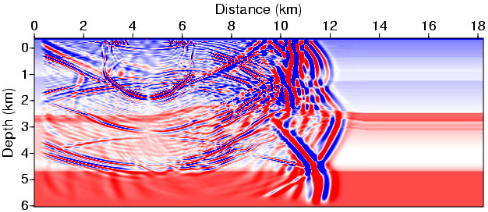

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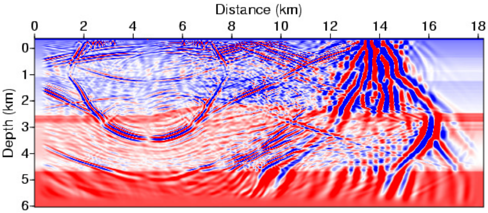

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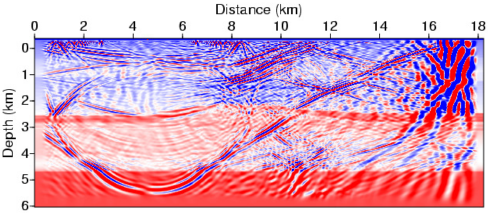

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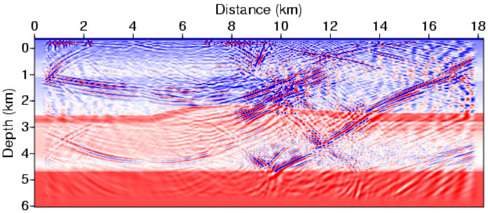

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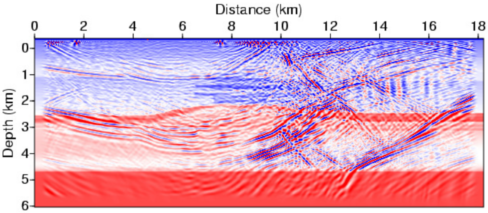

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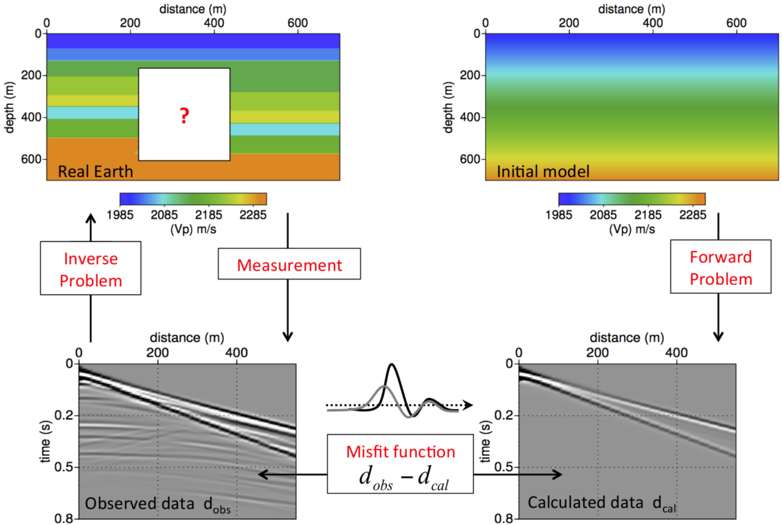

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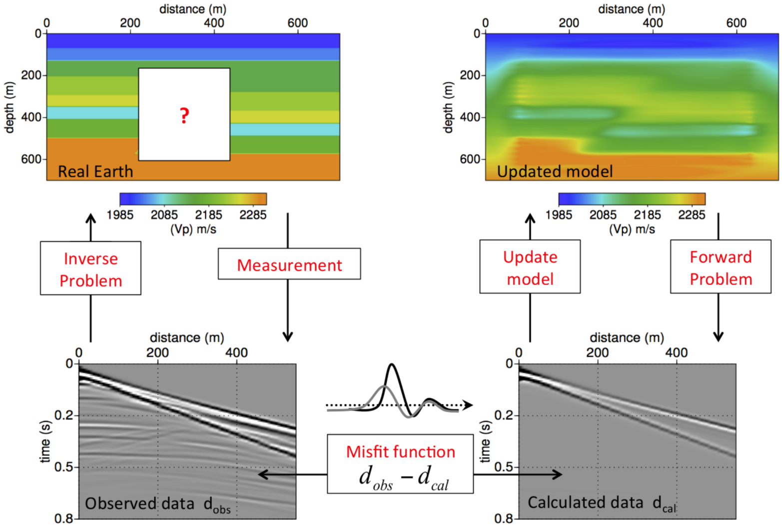

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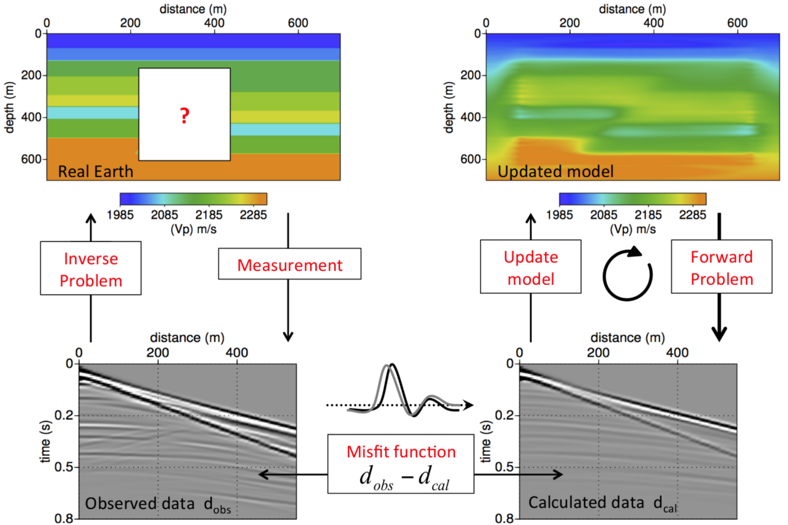

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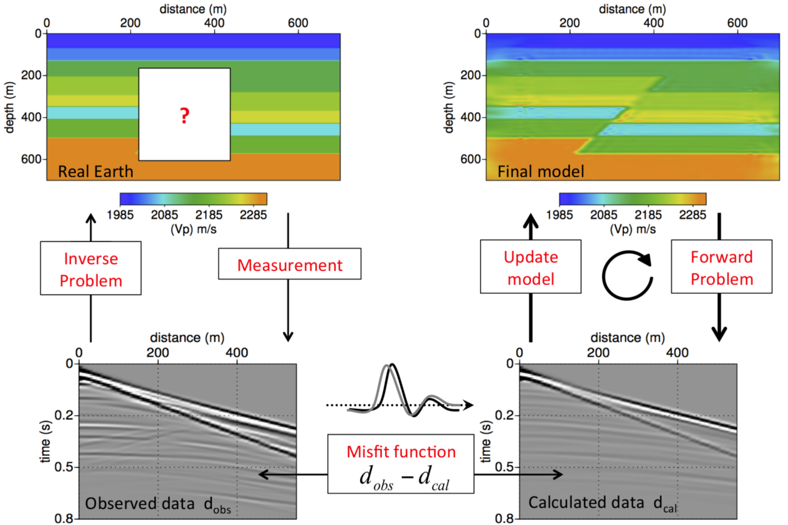

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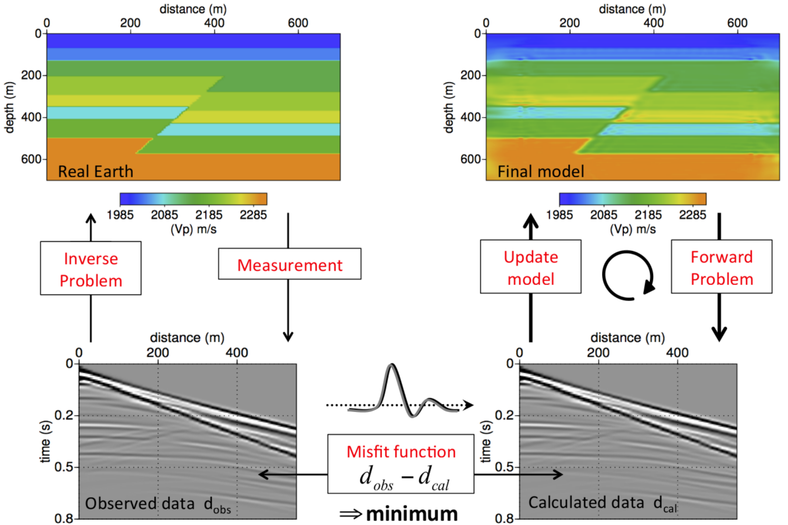
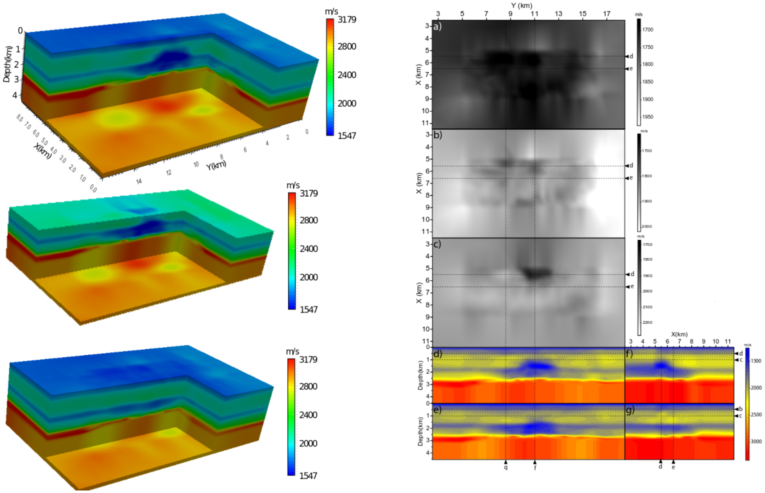
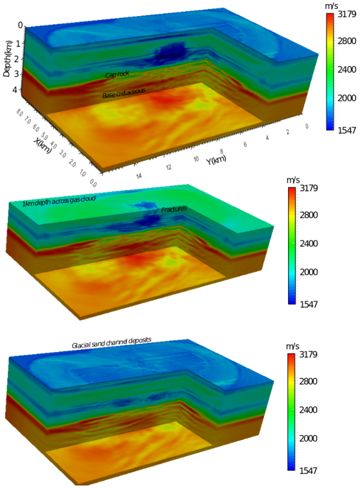
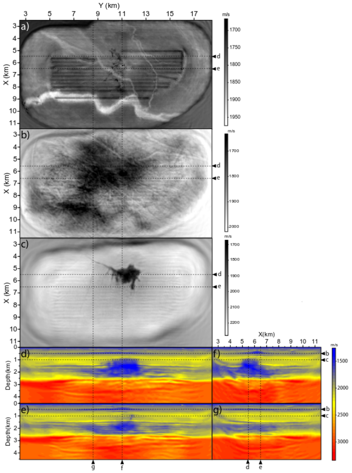




Traveltime tomography in the Valhall Oil Field.





Full Waveform Inversion in the Valhall Oil Field.

Motivations

- FWI is generally applied in a deterministic fashion from a starting model
- FWI relies on local optimization (quasi-Newton)
- FWI results are generally difficult to assess

Only a few recent papers propose to tackle the uncertainy problem in FWI : still no systematic applications.

We propose an approach relying on a mixed-method based on an Ensemble Transform Kalman Filter and the classic quasi-Newton optimization scheme to evaluate uncertainty in the FWI results.

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We define the FWI problem as a non-linear operator $\mathcal{F}$

$$\mathcal{F}(m) = \min_{m} \quad \frac{1}{2}||d_{cal}(m) - d_{obs}||^2$$

- m is the model containing the n physical parameters
- $d_{cal}(m)$ the synthetic wavefield data computed in m
- d_{obs} the observed data
- $||.||$ the Euclidean distance in the data space

Applying $\mathcal{F}$ on an initial model $m_0 \rightarrow$ unique solution with a local optimization scheme.

But how do we apply an EnKF to this static problem?

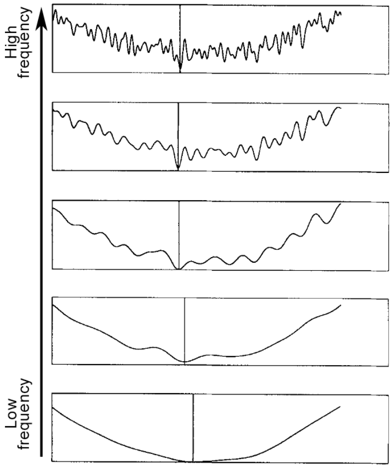
Multi Scale Approach

Full Waveform data fitting is **ill-posed** by nature.

Non-unique, its cost function can be strongly non-convex.

The cost function convexity is primarily dominated by the data frequency content (due to the nature of cycle skipping problem)

We can use the multi-scale frequency strategy as a proxy for evolution in the frequency domain.



1D waveform cost function, at different frequency content from (Bunks et al., 1995)

We can recast our problem as an ensemble representation. Our ensemble **m** is a collection of N_e models m^i, with $i = 1, 2, \ldots, N_e$.

In place of the typical DA forecast forward modeling problem we have :

$$m_k^{f\,i} = \mathcal{F}(m_{k-1}^i) \tag{1}$$

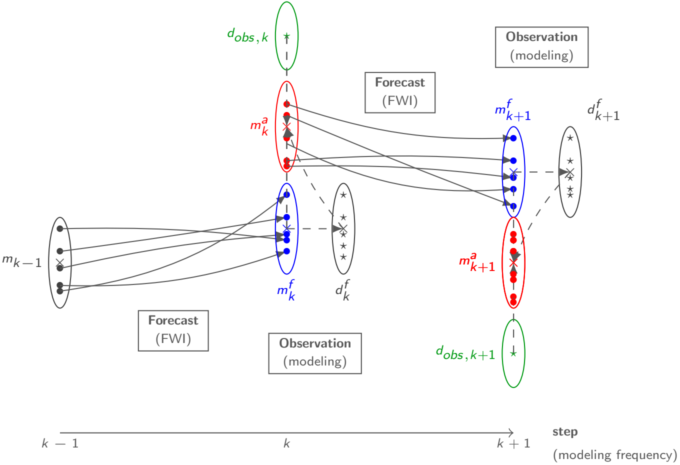
We decompose the d_{obs} in K frequency bands $\rightarrow$ solve FWI independently on each of N_e models, at a given frequency k.
Allowing to consider a dynamic axis in frequency, with $k = 1, \ldots, K$ instead of temporal evolution

Scheme specificities

The EnKF scheme we follow : the Ensemble Transform Kalman Filter (**ETKF**) (Bishop et al., 2001).

We apply FWI in the frequency domain → complex wavefield data.

We consider all measurements as uncorrelated → measurement noise operator is diagonal whose values are calibrated on the data noise level.



$d_{obs,k}$
Observation
(modeling)
Forecast
(FWI)
m^f_{k+1}
d^f_{k+1}
m^a_k
m_{k-1}
m^f_k
d^f_k
m^a_{k+1}
Forecast
(FWI)
Observation
(modeling)
$d_{obs,k+1}$
$k-1$
k
$k+1$
step
(modeling frequency)

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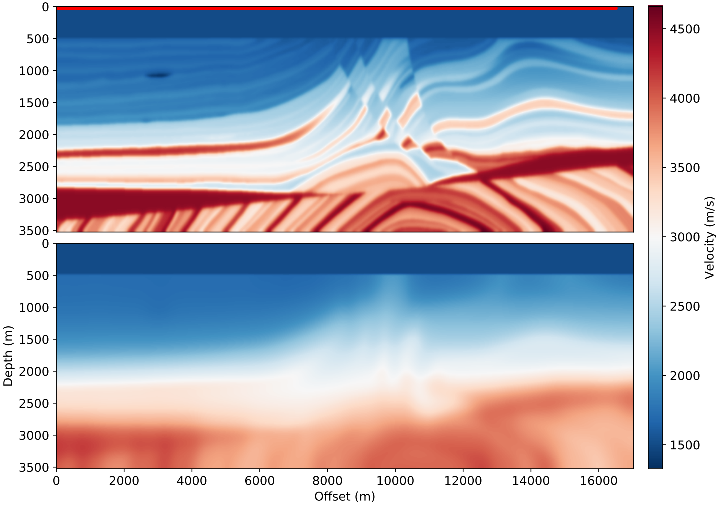
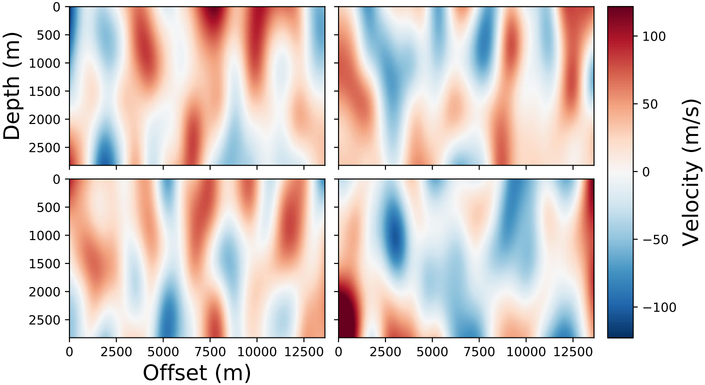
Application on 2D Marmousi model :

- Fixed spread surface acquisition (144 sources, 660 receivers)
- Noisy signal (SNR = 5)
- 15 ETKF-FWI cycles from 3 to 10Hz.
- Initial gaussian repartition

Applying ETKF to FWI



Numerical test setting. Top : True model, bottom : Initial model



Exemple of random perturbation selected to build the initial gaussian repartition.

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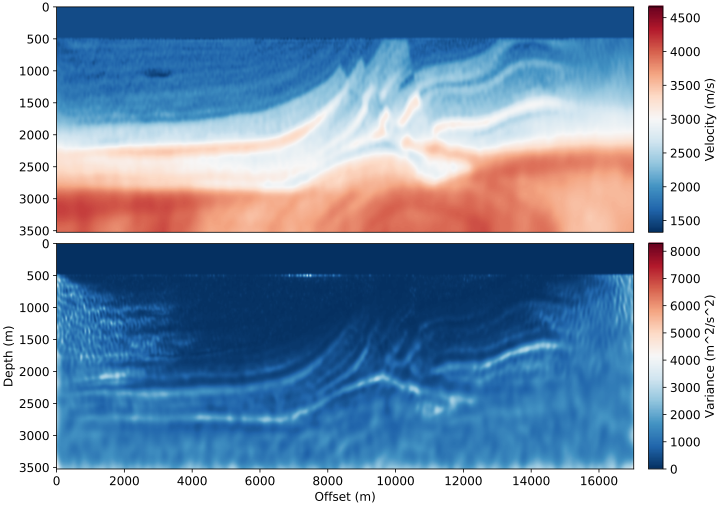
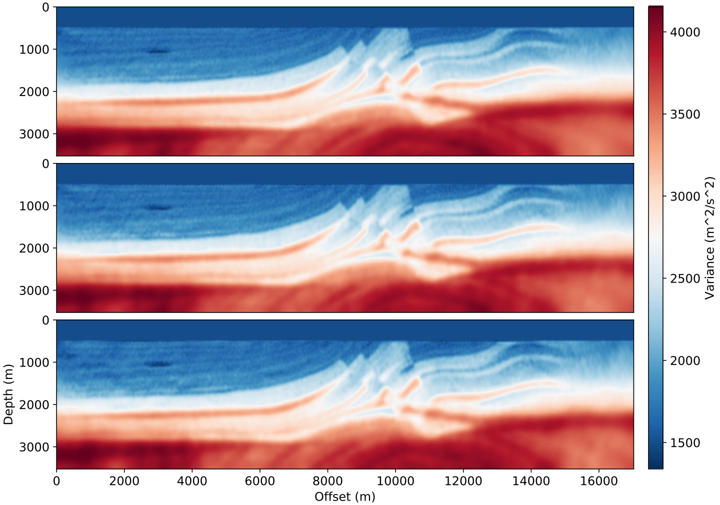
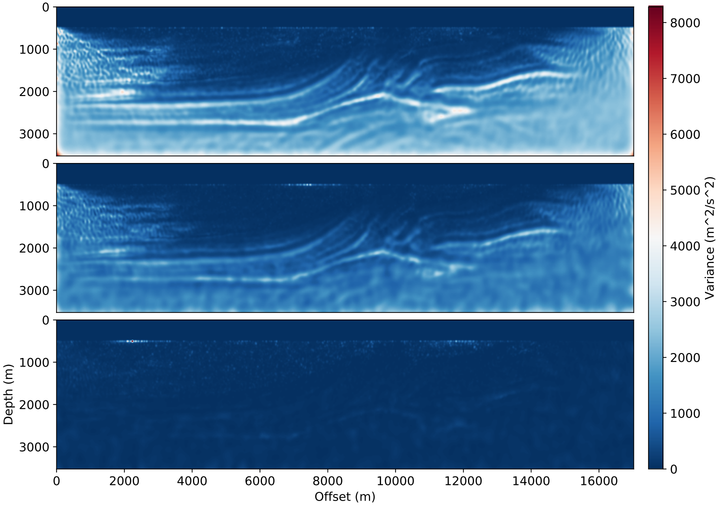


Results for 200 ensemble members. Top : Final ensemble mean, bottom : Final ensemble variance



Numerical test. Ensemble mean for $N_e = 2000$, 200 and 20

Undersampling sensitivity - Ensemble variance



Numerical test. Variance for $N_e = 2000$, 200 and 20

A wide range of variance values

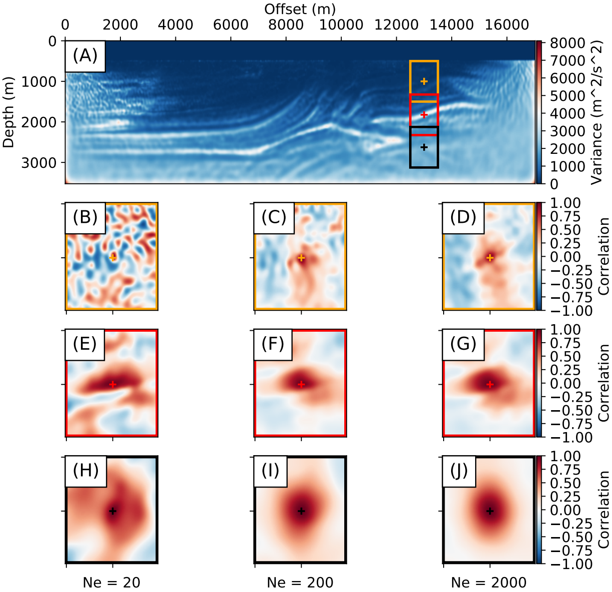
That high degree of variability makes qualitative comparison of off-diagonal terms difficult.

We propose to use the Correlation matrix $\mathcal{C}_k^a$ instead of the Covariance matrix to read the off-diagonal terms.

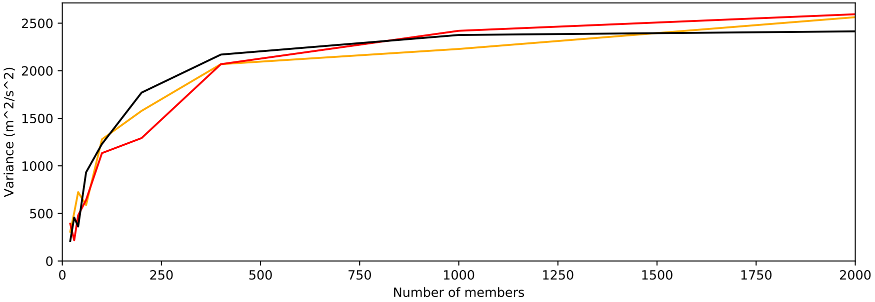
$$\mathcal{C}_{k,e}^a = (diag(\mathbf{P}_{k,e}^a))^{-1/2}\mathbf{P}_{k,e}^a(diag(\mathbf{P}_{k,e}^a))^{-1/2} \qquad (2)$$

This provides dimensionless and normalized correlation maps, regardless of N_e and location in the medium.

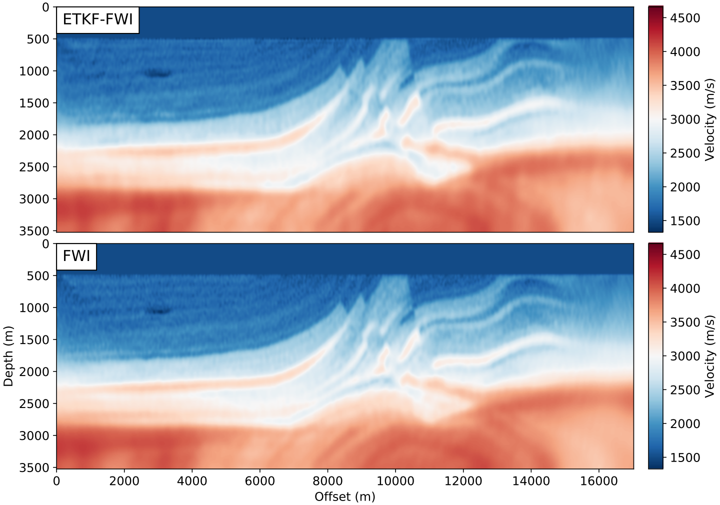
Estimating local correlation maps



Variance convergence test

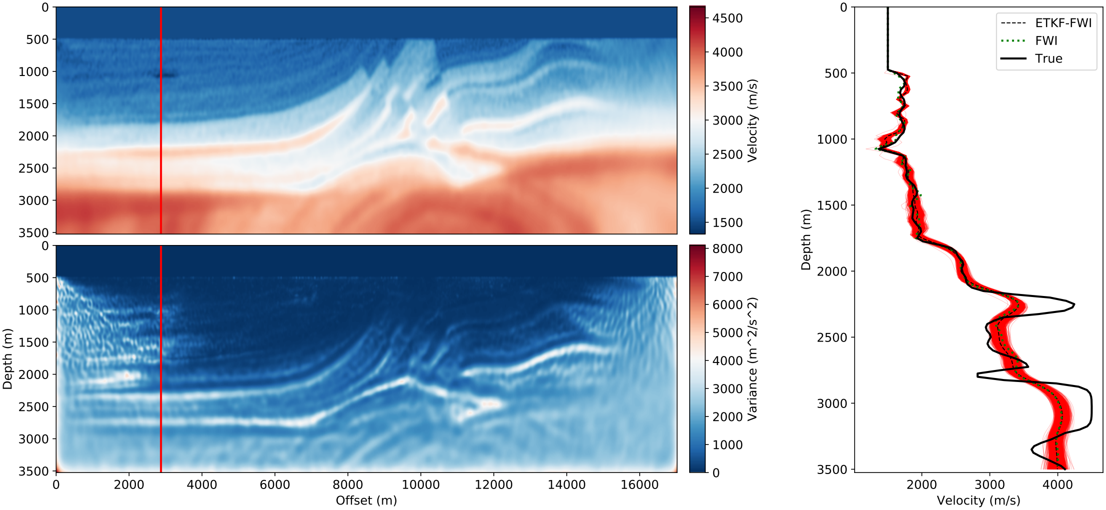


Influence of N_e on variance estimation



Top : Ensemble Mean. Bottom : FWI result

Velocity Log - Comparing ETKF-FWI with FWI



Velocity log through the $N_e = 2000$ ensemble.

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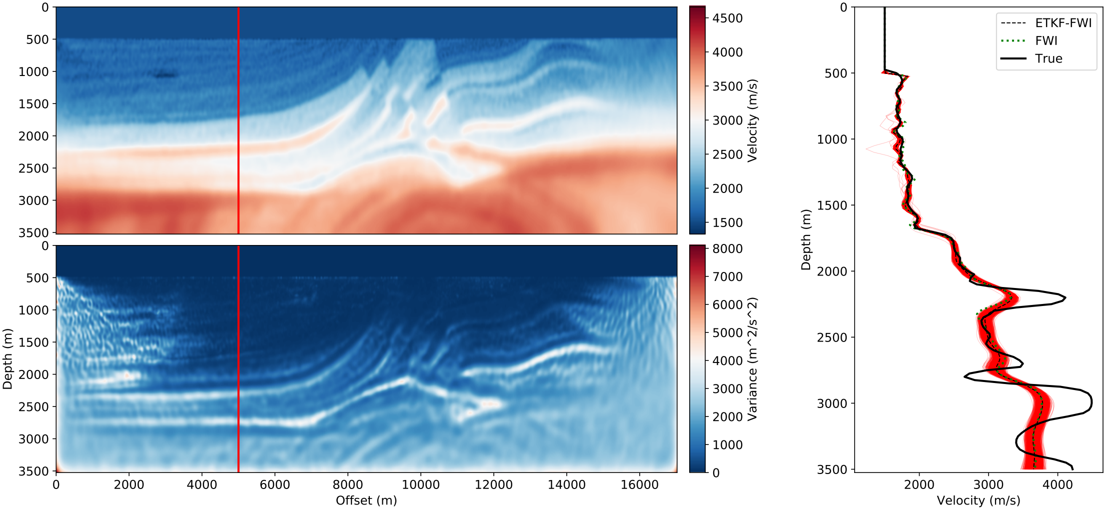

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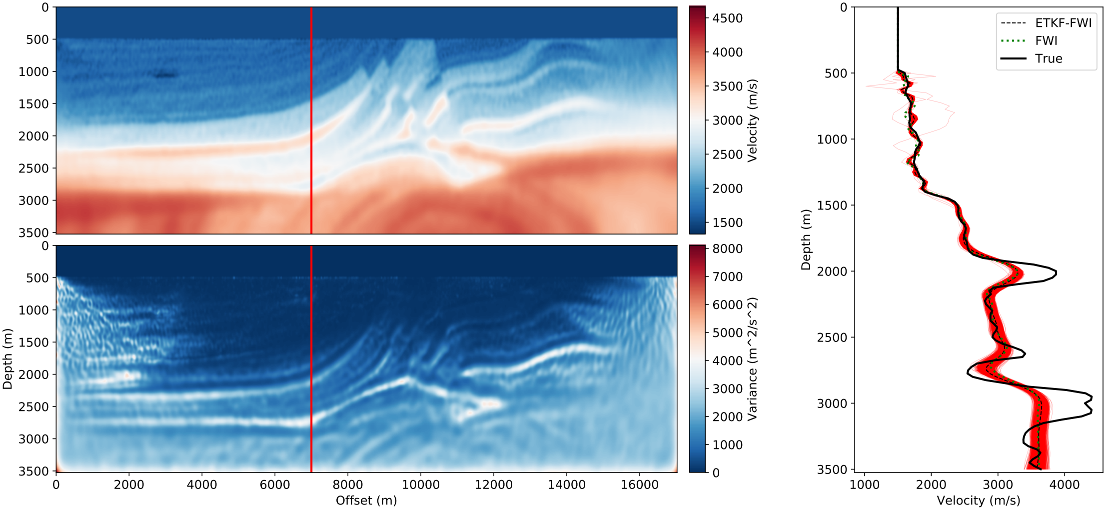

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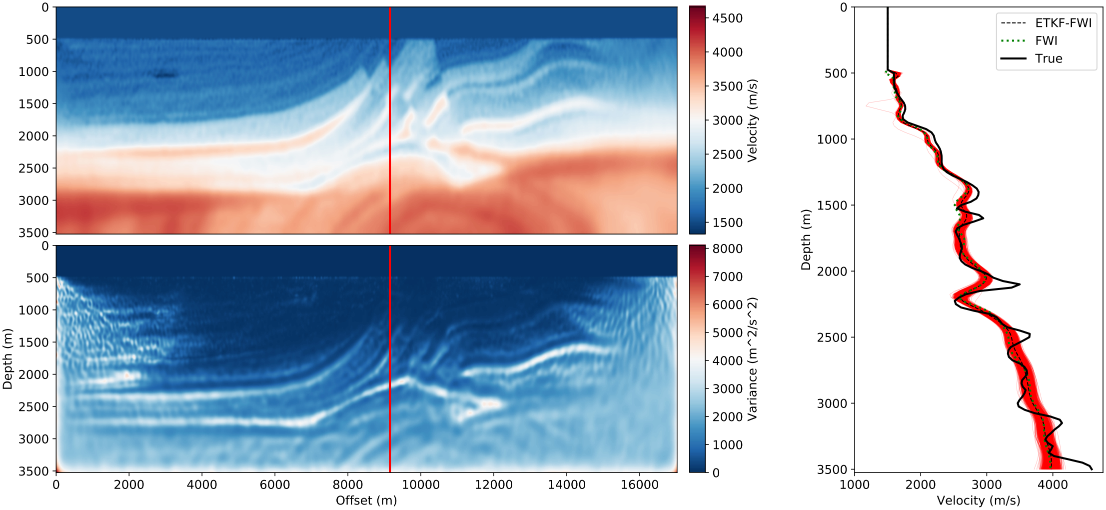

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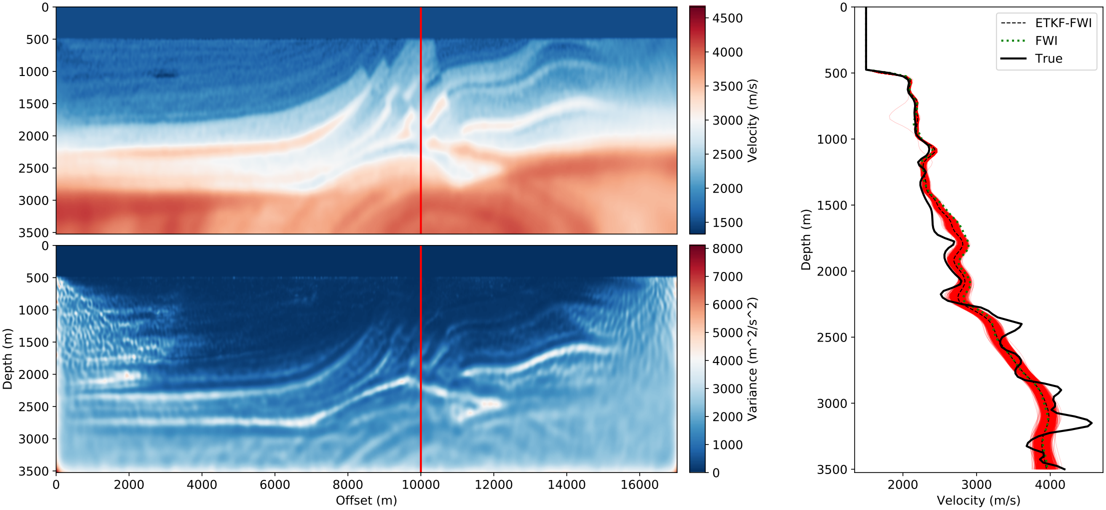

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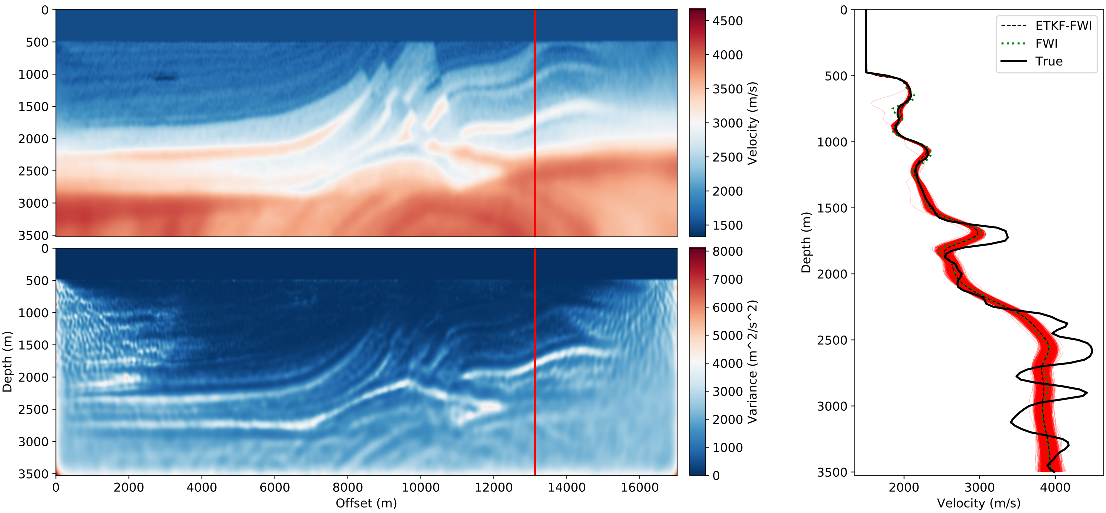

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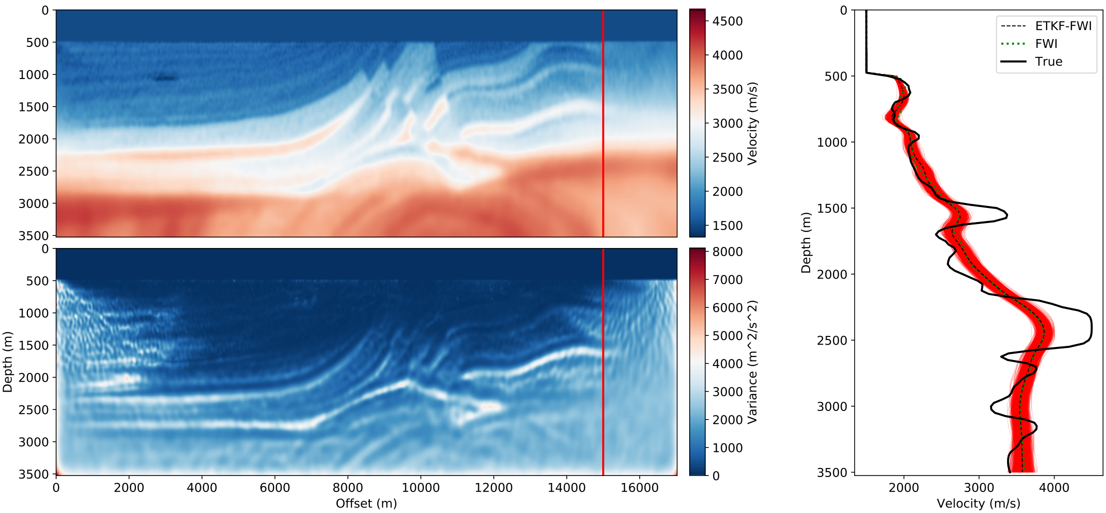

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Uncertainty estimation is possible with ETKF-FWI.

Numerical experiments show:

- Very low N_e = dramatic underestimation of P_e^a
- Higher N_e = stable approximation
- Variance underestimation = power-low trend
- Mean is preserved
- Possible local "collapse" : strong undersapling in shallow zones

Uncertainty estimation not absolute uncertainty.

Perspectives

Short term :

- Undersampling mitigation (inflation tests in progress).
- Initial ensemble building.

Medium term :

- Real data application : Valhall 2D.
- Comparison with other methodologies.

Long term prospective work:

- Go beyond 2D frequency acoustic (3D, time domain, multiparameter...)
- Inversion parameters influence over uncertainty
- Sensor fusion (well log data, geophysical methods)

Thanks for your attention

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- CIMENT (Froggy) computing center *https://ciment.ujf-grenoble.fr*
- CINES/IDRIS/TGCC computing center (allocation 046091 made by GENCI)

Questions?

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